

Matter Enhancement of T Violation in Neutrino Oscillation

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– Abstract –

We study the matter enhancement of T violation in neutrino oscillation with three generations. The magnitude of T violation is proportional to Jarlskog factor J . Recently, the elegant relation, $(\Delta_m)_{12}(\Delta_m)_{23}(\Delta_m)_{31}J_m = \Delta_{12}\Delta_{23}\Delta_{31}J$, was derived, where $\Delta_{ij} = \Delta m_{ij}^2/(2E)$ and subscript m implies the quantities in matter. Using this relation, we reconsider how J_m changes as a function of the matter potential a under the approximation $|\Delta m_{12}^2| \ll |\Delta m_{13}^2|$. We show that the number of maxima for J_m depends on the magnitude of $\sin^2 2\theta_{13}$ and there are two maxima considering the constraint on $\sin^2 2\theta_{13}$ from the CHOOZ experiment. One maximum of J_m at $a = O(\Delta_{12})$ is given by $J/\sin 2\theta_{12}$, which leads to the large enhancement of J_m in the case of the SMA MSW solution. The other maximum at $a = O(\Delta_{13})$ is $|\Delta_{12}/\Delta_{13}|J/\sin 2\theta_{13}$, and the enhancement is possible, if $\sin 2\theta_{13}$ is small enough. These maximal values are consistent with the results obtained by other methods.

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1 Introduction

Solar neutrino experiments have been observing a ν_e deficit for a long time [1] and the ratio of ν_μ/ν_e in atmospheric neutrino has implied a ν_μ deficit [2], which are explained by ν_e - ν_μ oscillation and ν_μ - ν_τ oscillation, respectively. These experiments provide strong evidence that there exist masses and mixings in the lepton sector with three generations [3].

Long baseline experiments [4] and neutrino factories [5] are operated, or planned, in order to obtain more convincing evidence for neutrino oscillation. Furthermore it could also be possible to observe CP and T violations.

As the neutrinos pass through the earth in these experiments, matter effects must be considered. It has been studied in the context of long baseline experiments [6], and in the context of a neutrino factory [7]. T violation is different from CP violation in matter and it is pointed out that it is easy to calculate T violation compared with CP violation for neutrino oscillation in matter [8]. The T violating part in matter, $\Delta P_T = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$, ($\alpha, \beta = e, \mu, \tau$) is proportional to Jarlskog factor J_m [9] of the lepton sector, unlike the CP violating part. The dependence of J_m on the matter potential $a = \sqrt{2}G_F N_e$ is investigated in other works [10, 11].

Recently, Harrison and Scott [12] derived the relation

$$(\Delta_m)_{12}(\Delta_m)_{23}(\Delta_m)_{31}J_m = \Delta_{12}\Delta_{23}\Delta_{31}J, \quad (1)$$

where $\Delta_{ij} = \Delta m_{ij}^2/(2E)$ and the quantities with the subscript m are those in matter. The inverse of J_m is the square root of a quartic function of a . This means that J_m has either one or two local maxima as a function of a .

In this letter, we present both the exact and approximate form of J_m as a function of a using the above relation. It is shown that the number of resonant maxima of J_m depends on the magnitude of $\sin^2 2\theta_{13}$. Taking account of the constraint on $\sin^2 2\theta_{13}$ from the CHOOZ experiment [13], we show that there exist two maxima. We also estimate the maximal values of J_m in the cases of small mixing angle (SMA) and large mixing angle (LMA) MSW solutions [14].

2 T Violation in Neutrino Oscillation

We review T violation in three-neutrino oscillations and state the strategy of this letter. In vacuum, flavor eigenstates ν_α ($\alpha = e, \mu, \tau$) are related to mass eigenstates ν_i ($i = 1, 2, 3$), which have the mass eigenvalues m_i , by the unitary transformation,

$$\nu_\alpha = U_{\alpha i}\nu_i, \quad (2)$$

where $U_{\alpha i}$ is the Maki-Nakagawa-Sakata matrix [3]. The T violating part, $\Delta P_T(\nu_\alpha \rightarrow \nu_\beta) \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$, in three generation after traveling a distance L is calculated as

$$\Delta P_T = 16J \sin \frac{\Delta_{12}L}{2} \sin \frac{\Delta_{23}L}{2} \sin \frac{\Delta_{31}L}{2}, \quad (3)$$

where

$$J \equiv \text{Im}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}]. \quad (4)$$

In order to obtain the T violating part in matter, we only have to replace $\Delta_{ij} \rightarrow (\Delta_m)_{ij}$, $U_{\alpha i} \rightarrow (U_m)_{\alpha i}$, hence, $J \rightarrow J_m$.

We would like to study the case where large ΔP_T is realized. In eq. (3), ΔP_T is a product of J_m and trigonometric functions. In the following calculation, we focus on the matter effect of J_m which does not depend on L and determine the maxima of J_m .

As seen in eq. (4), J consists of the product of $U_{\alpha i}$. It is complicated to calculate J_m directly from $(U_m)_{\alpha i}$, which diagonalizes the matter-modified Hamiltonian H_m , although the numerical calculation has been performed [10]. However, it is possible to calculate J_m without direct calculation of $(U_m)_{\alpha i}$ from the relation

$$(\Delta_m)_{12}(\Delta_m)_{23}(\Delta_m)_{31}J_m = \Delta_{12}\Delta_{23}\Delta_{31}J, \quad (5)$$

derived by Harrison and Scott [12]. Since the right hand-side of eq. (5) is a constant which does not depend on the matter effect, J_m is inversely proportional to a triple product of $(\Delta_m)_{ij}$. Therefore, we study the function of the matter potential a such as

$$f(a) \equiv [(\Delta_m)_{12}(\Delta_m)_{23}(\Delta_m)_{31}]^2, \quad (6)$$

and determine the minima of $f(a)$.

3 Triple Product of Mass Square Differences

In this section, we study the matter effect on $f(a)$. Harrison and Scott [12] suggest that $f(a)$ is a quartic function of the matter potential a and in principle its coefficients can be written by the parameters Δm_{ij}^2 and $U_{\alpha i}$ in vacuum, although it is complicated in practice. We present the exact form of $f(a)$ in relatively simple form by introducing new parameters. The coefficients of $f(a)$ is further simplified under the approximation $|\Delta m_{12}^2| \ll |\Delta m_{13}^2|$.

First, let us note that $(\Delta_m)_{ij}$ included in $f(a)$ are rewritten by the eigenvalues $(\lambda_m)_i$ of the matter-modified Hamiltonian H_m as $(\Delta_m)_{ij} = (\lambda_m)_j - (\lambda_m)_i$. The eigenvalues $(\lambda_m)_i$ are the solutions of the equation for t ,

$$\det(H_m - t) = (\lambda_1 - t)(\lambda_2 - t)(\lambda_3 - t) + a(t - \delta_2)(t - \delta_3) = 0, \quad (7)$$

where δ_i ($i = 2, 3$) and λ_i ($i = 1, 2, 3$) are the eigenvalues of the 2×2 submatrix H_{ij} ($i, j = 2, 3$) and 3×3 matrix H in vacuum. After a calculation, $f(a)$ is expressed as a quartic function of a :

$$f(a) = [((\lambda_m)_2 - (\lambda_m)_1)((\lambda_m)_3 - (\lambda_m)_2)((\lambda_m)_1 - (\lambda_m)_3)]^2 \quad (8)$$

$$= f_4 a^4 + f_3 a^3 + f_2 a^2 + f_1 a + f_0, \quad (9)$$

where the coefficients f_i ($i = 0, \dots, 4$) are presented by λ_i and δ_i in the following.

The coefficients f_4 and f_0 are

$$f_4 = (\delta_2 - \delta_3)^2, \quad (10)$$

$$f_0 = \{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_1 - \lambda_3)\}^2. \quad (11)$$

By definition (6), $f(a)$ is semi-positive definite, hence, $f_4, f_0 \geq 0$ must be satisfied taking account of the limit $a \rightarrow \infty$ and $a \rightarrow 0$. The relations (10) and (11) are consistent with these conditions.

The other coefficients are

$$f_3 = 2[(\delta_2 - \lambda_1)(\delta_2 - \lambda_2)(\delta_2 - \lambda_3) + (\delta_3 - \lambda_1)(\delta_3 - \lambda_2)(\delta_3 - \lambda_3)] \\ - 2(\delta_2 - \delta_3)^2[(\delta_2 - \lambda_1) + (\delta_3 - \lambda_1) + (\text{cyclic of } \lambda_i)], \quad (12)$$

$$f_2 = [(\delta_2 - \lambda_1)(\delta_3 - \lambda_2) + (\text{cyclic of } \lambda_i)]^2 \\ - 6[(\delta_2 - \lambda_1)(\delta_2 - \lambda_2)\{(\delta_3 - \lambda_1) + (\delta_3 - \lambda_2)\}(\delta_3 - \lambda_3) + (\text{cyclic of } \lambda_i)], \quad (13)$$

$$f_1 = 4[(\delta_2 - \lambda_2)(\delta_3 - \lambda_2)(\lambda_1 - \lambda_3)^2(\lambda_2 - \lambda_1) + (\text{cyclic of } \lambda_i)] \\ + 2(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_1 - \lambda_3)[(\delta_2 - \lambda_1)(\delta_3 - \lambda_2) + (\text{cyclic of } \lambda_i)], \quad (14)$$

which are relatively simple compared with the case where we don't introduce new parameters δ_i . In section 5, we present the figures using these coefficients.

Next, let us show that these coefficients are further simplified under the approximation $|\Delta_{12}| \ll |\Delta_{13}|$. As δ_i are the eigenvalues of submatrix

$$\begin{pmatrix} H_{22} & H_{23} \\ H_{32} & H_{33} \end{pmatrix} = \lambda_1 \mathbf{1} + \Delta_{13} \begin{pmatrix} |U_{\mu 3}|^2 & U_{\mu 3} U_{\tau 3}^* \\ U_{\tau 3} U_{\mu 3}^* & |U_{\tau 3}|^2 \end{pmatrix} + \Delta_{12} \begin{pmatrix} |U_{\mu 2}|^2 & U_{\mu 2} U_{\tau 2}^* \\ U_{\tau 2} U_{\mu 2}^* & |U_{\tau 2}|^2 \end{pmatrix}, \quad (15)$$

where $\mathbf{1}$ is the unit matrix, they are approximated by

$$\delta_2 = \lambda_1 + \frac{|U_{e1}|^2 \Delta_{12}}{1 - |U_{e3}|^2}, \quad \delta_3 = \lambda_1 + (1 - |U_{e3}|^2) \Delta_{13} + \frac{|U_{e2}|^2 |U_{e3}|^2 \Delta_{12}}{1 - |U_{e3}|^2}, \quad (16)$$

up to the first order of Δ_{12} using the unitarity condition. Substituting eq. (16) for δ_i in eqs. (10)~(14) and taking the standard parameterization, $U_{e1} = c_{12}c_{13}$, $U_{e2} = s_{12}c_{13}$, $U_{e3} = s_{13}e^{-i\delta}$, where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, the coefficients are calculated as

$$f_4 \simeq c_{13}^4 (\Delta_{13})^2, \quad f_3 \simeq -2c_{13}^4 \cos 2\theta_{13} (\Delta_{13})^3, \quad f_2 \simeq c_{13}^4 (\Delta_{13})^4, \\ f_1 \simeq -2c_{13}^2 \cos 2\theta_{12} \Delta_{12} (\Delta_{13})^4, \quad f_0 \simeq (\Delta_{12})^2 (\Delta_{13})^4, \quad (17)$$

at the leading order. Note that the order of Δ_{12} for f_i is important when we determine the minima of $f(a)$. f_1 is the first order of Δ_{12} and f_2, f_3, f_4 are the zeroth order. Its difference determines the magnitude of a for each minima.

4 Matter Enhancement of the Jarlskog Factor

In this section, we calculate the minima of $f(a)$ using the coefficients (17) in order to determine the maxima of J_m . First, we show that the number of minima depends

on the magnitude of $\sin^2 2\theta_{13}$, and that there are two minima taking account of the constraint on $\sin^2 2\theta_{13}$ from the CHOOZ experiment. Second, we estimate the maximal values of J_m and the energies of the neutrino at maxima in the cases of the SMA and LMA MSW solutions.

Let us start with differentiating $f(a)$ in terms of a :

$$f(a)' = 4f_4a^3 + 3f_3a^2 + 2f_2a + f_1 = 0. \quad (18)$$

Since only f_1 is $O(\Delta_{12})$ in $f_i (i = 1, 2, 3, 4)$ from eq. (17), in the limit of $\Delta_{12} \rightarrow 0$, eq. (18) reduces to

$$a(4f_4a^2 + 3f_3a + 2f_2) = 0. \quad (19)$$

Hence, there exists a solution at $a = 0$ in this limit. This means that a solution at $a = O(\Delta_{12})$ exists for $\Delta_{12} \neq 0$.

On the other hand, whether another minimum exists or not is determined by the discriminant D of the quadratic equation in the parenthesis of eq. (19),

$$D = 9f_3^2 - 32f_4f_2 = 4c_{13}^8(\Delta_{13})^{12}(1 - 9\sin^2 2\theta_{13}). \quad (20)$$

If $\sin^2 2\theta_{13} > 1/9$, there exists only one minimum at $a = O(\Delta_{12})$ as Fig. 1 (a). If $\sin^2 2\theta_{13} < 1/9$, then there exists another minimum at $a = O(\Delta_{13})$ as Fig. 1 (b). The restriction of the CHOOZ experiment, $\sin^2 2\theta_{13} \leq 0.10$ [13], is included in the case of Fig. 1 (b).

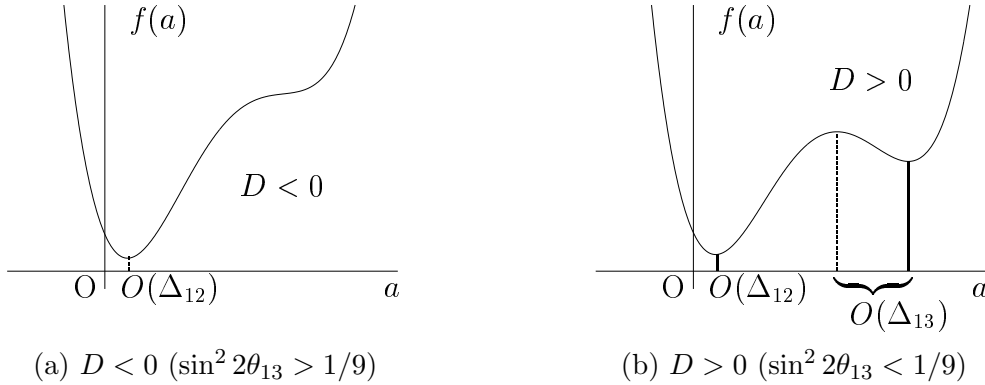


Fig. 1. $f(a)$ has two(one) local minima for $D > 0$ ($D < 0$). The CHOOZ experiment favors $D > 0$ and the figure (b).

(I) The maximal value of J_m at $a = O(\Delta_{12})$

The solution of eq. (18) at $a = O(\Delta_{12})$ is

$$a = \frac{\cos 2\theta_{12}}{\cos^2 \theta_{13}} \Delta_{12}. \quad (21)$$

The minimal value is

$$f(a) = \sin^2 2\theta_{12}(\Delta_{12})^2(\Delta_{13})^4, \quad (22)$$

and thus, from eq. (5), the maximum of the ratio is given by

$$\frac{J_m}{J} = \frac{1}{\sin 2\theta_{12}}, \quad (23)$$

which is consistent with other works [10, 11]. This means that J_m is largely enhanced in the case of the SMA MSW solution.

We estimate J_m/J in two MSW solutions:

$$\frac{J_m}{J} = \begin{cases} 12, & \text{for SMA MSW,} \\ 1.1, & \text{for LMA MSW,} \end{cases} \quad (24)$$

where we use $\sin^2 2\theta_{12} = 7.2 \times 10^{-3}$ (SMA MSW), 0.79 (LMA MSW) [15].

The neutrino energy corresponding to the maximum of J_m/J , from eq. (21), is

$$E = \frac{\cos 2\theta_{12} \Delta m_{12}^2}{2\sqrt{2}G_F N_e c_{13}^2}, \quad (25)$$

where N_e is the electron number density: $N_e = 8.2 \times 10^{23} \text{cm}^{-3}$ in the earth's crust. Substituting the experimental data, it is obtained as

$$E = \begin{cases} 25 \text{ MeV,} & \text{for SMA MSW,} \\ 62 \text{ MeV,} & \text{for LMA MSW,} \end{cases} \quad (26)$$

where we use $\Delta m_{12}^2 = 5.0 \times 10^{-6}$ (SMA MSW), 2.7×10^{-5} (LMA MSW), and $\sin^2 2\theta_{13} = 0.10$ (the upper limit of the CHOOZ experiment).

(II) The maximal value of J_m at $a = O(\Delta_{13})$

The other solutions of eq. (18) at $a = O(\Delta_{13})$ are

$$a = \frac{1}{4} \left(3 \cos 2\theta_{13} \pm \sqrt{1 - 9 \sin^2 2\theta_{13}} \right) \Delta_{13}, \quad (27)$$

where the sign $+$ for $\cos 2\theta_{13} \geq 0$ and the sign $-$ for $\cos 2\theta_{13} \leq 0$. The minimal value is

$$f(a) = \frac{c_{13}^4 (\Delta_{13})^6}{32} \left[4 - 3(1 - 3 \sin^2 2\theta_{13})^2 - \cos 2\theta_{13} (1 - 9 \sin^2 2\theta_{13})^{\frac{3}{2}} \right] \quad (28)$$

and the maximum of the ratio is given by

$$\frac{J_m}{J} = \left| \frac{\Delta_{12}}{\Delta_{13}} \right| \frac{1}{c_{13}^2} \frac{4\sqrt{2}}{\sqrt{4 - 3(1 - 3 \sin^2 2\theta_{13})^2 - \cos 2\theta_{13} (1 - 9 \sin^2 2\theta_{13})^{\frac{3}{2}}}}. \quad (29)$$

Because of the suppression factor $|\Delta_{12}/\Delta_{13}|$, the enhancement of J_m is small compared with the case (I).

Furthermore, we can obtain more simple forms for eq. (27) and eq. (29) under the approximation $9 \sin^2 2\theta_{13} \ll 1$, although this approximation is not justified near the

upper limit, $\sin^2 2\theta_{13} \simeq 0.10$, of the CHOOZ experiment. In this case, the value of a for the maximum of J_m is

$$a = \left(1 - \frac{3}{2} \sin^2 2\theta_{13}\right) \Delta_{13} \quad (30)$$

and the ratio is

$$\frac{J_m}{J} = \left| \frac{\Delta_{12}}{\Delta_{13}} \right| \frac{1}{\sin 2\theta_{13}}. \quad (31)$$

It is understood from this result that the enhancement of J_m is not always realized because of the suppression factor $|\Delta_{12}/\Delta_{13}|$. However, we still have an enhancement for small $\sin 2\theta_{13}$. For example, at $\sin^2 2\theta_{13} = 4.0 \times 10^{-6}$ which corresponds to $\sin \theta_{13} = 1.0 \times 10^{-3}$ and which is much smaller than the present upper limit, the maximum of the ratio is given by

$$\frac{J_m}{J} = \begin{cases} 0.78, & \text{for SMA MSW,} \\ 4.2, & \text{for LMA MSW,} \end{cases} \quad (32)$$

where we use the experimental values for $\Delta m_{23}^2 = 3.2 \times 10^{-3} \text{eV}^2$. Thus, J_m for the LMA MSW solution has an enhancement which is several times as large as J in this example.

The neutrino energy corresponding to the maximum of J_m is calculated from eq. (30),

$$E = \left(1 - \frac{3}{2} \sin^2 2\theta_{13}\right) \frac{\Delta m_{13}^2}{2\sqrt{2}G_F N_e}. \quad (33)$$

Substituting the same experimental data as before we obtain

$$E = 16 \text{ GeV}. \quad (34)$$

We summarize the above two maxima of J_m in Table 1.

a	J_m/J and E		SMA MSW	LMA MSW
$O(\Delta_{12})$	J_m/J	$1/\sin 2\theta_{12}$	12	1.1
	E	$\cos 2\theta_{12} \Delta m_{12}^2 / (2\sqrt{2}G_F N_e c_{13}^2)$	24MeV	60MeV
$O(\Delta_{13})$	J_m/J	$\Delta m_{12}^2 / (\Delta m_{13}^2 \sin 2\theta_{13})$	0.78	4.2
	E	$[1 - (3/2) \sin^2 2\theta_{13}] \Delta m_{13}^2 / (2\sqrt{2}G_F N_e)$	16GeV	16GeV

Table 1: The maxima of J_m/J and the neutrino energies E . Input parameters for vacuum are the same as in the text, and $\sin^2 2\theta_{13} = 4.0 \times 10^{-6}$ is chosen.

5 Numerical Estimation of the Ratio J_m/J

In this section, we numerically study the dependence of the ratio J_m/J on the neutrino energy E using eqs. (9)~(15). First, we illustrate the magnitude of maximum for J_m taking account of $\sin^2 2\theta_{12}$ and Δm_{12}^2 given by two MSW solutions and the

constraint on $\sin^2 2\theta_{13}$ from the CHOOZ experiment. Second, we study the effect of the signs of Δ_{12} and/or Δ_{13} . In the following calculation, input parameters are the same as in the previous section and we restrict to the range $0 < \theta_{ij} \leq \pi/4$ for simplicity.

Let us show the energy E dependence of J_m/J in the cases of the SMA and LMA MSW solutions with $\sin \theta_{13} = |U_{e3}| = 0.16$ and $|U_{e3}| = 1.0 \times 10^{-3}$ in Fig. 2 (a)~(d) ¹.

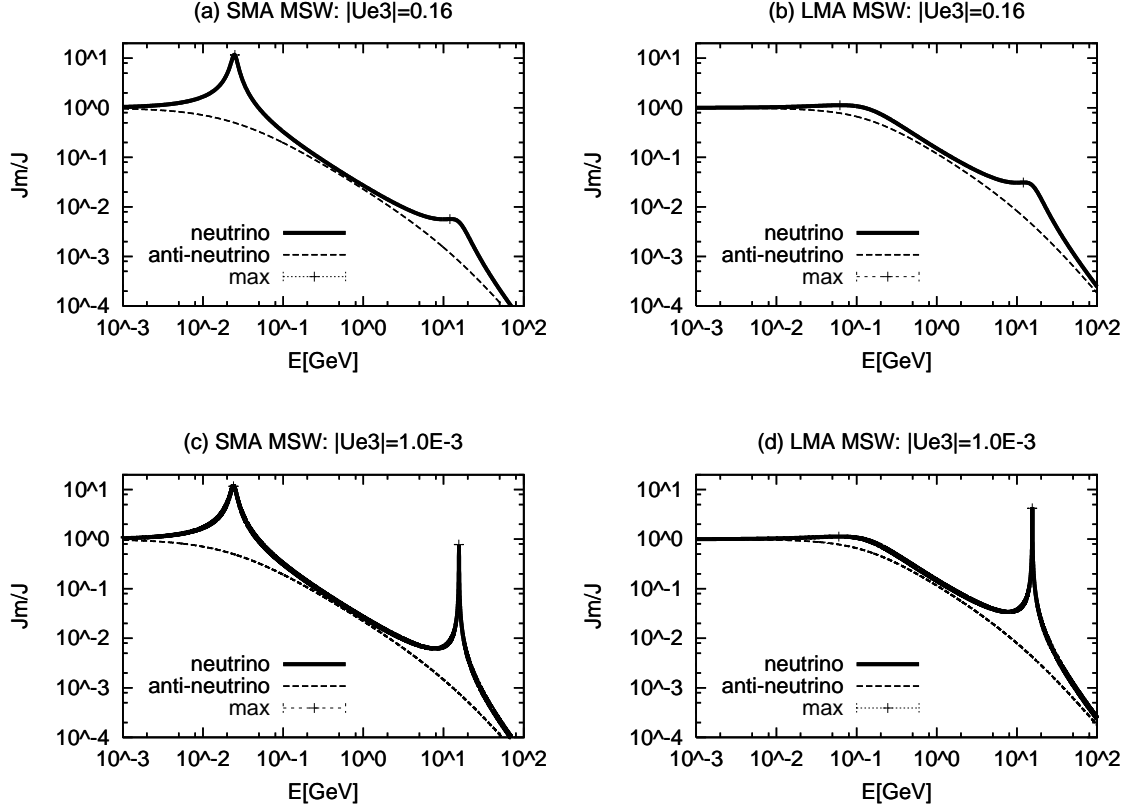


Fig. 2. The neutrino energy E dependence of J_m/J in the cases of the SMA and LMA MSW solutions with $|U_{e3}| = 0.16$ and $|U_{e3}| = 1.0 \times 10^{-3}$. The symbol + denotes the maxima determined in the previous section.

Comparing Fig. 2 (a) with (b) (or Fig. 2 (c) with (d)), we conclude that the SMA MSW solution has larger enhancement than the LMA MSW solution has for the maximum of J_m/J at $E = O(10\text{MeV})$. The enhancement, $J_m/J > 1$, occurs in the wide energy region around this maximum and the values calculated numerically almost coincide with the results (24) and (26) obtained approximately. Next, comparing Fig. 2 (a) and (c) (or Fig. 2 (b) and (d)), we conclude that if $\sin \theta_{13}$ is small enough, the enhancement for the maximum of J_m/J at $E = O(10\text{GeV})$ is possible although the energy region is small.

Next, we study the cases where Δm_{12}^2 and/or Δm_{13}^2 is negative. Since we have implicitly assumed that both Δm_{12}^2 and Δm_{13}^2 are positive until now, two maxima appear in “neutrino” oscillation. However, exactly speaking, whether the maxima appear in “neutrino” oscillation or “anti-neutrino” oscillation depends on the signs of

¹The energy dependence of J_m/J for a LMA MSW solution with $|U_{e3}|=0.090$ (corresponding to Fig. 2(b)) is shown in ref.[10].

Δm_{12}^2 and Δm_{13}^2 . In order to examine such cases, we numerically calculate J_m/J in the cases where Δm_{12}^2 and Δm_{13}^2 respectively, are positive and/or negative, and show the results for the SMA MSW solution as an example in Fig. 3.

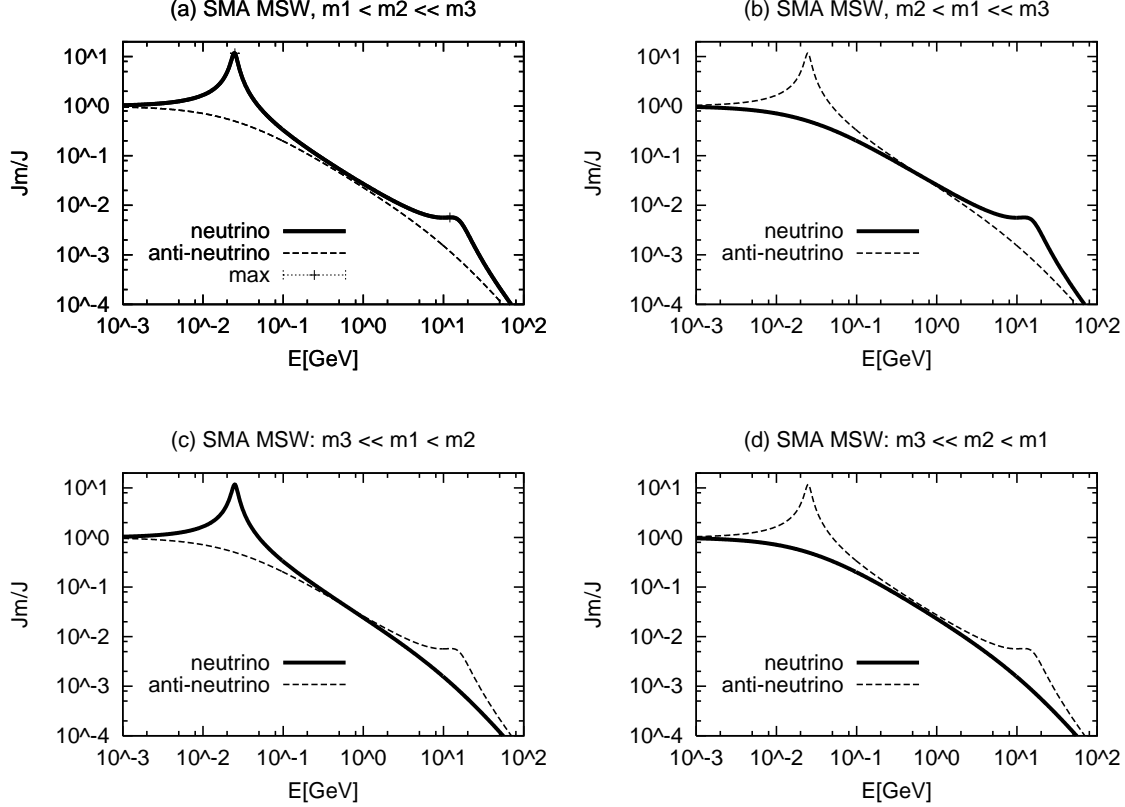


Fig. 3. The dependence of J_m on the sign of Δm_{ij}^2 . (a) and (c) are for $\Delta m_{12}^2 > 0$, (b) and (d) for $\Delta m_{12}^2 < 0$. (a) and (b) are for $\Delta m_{13}^2 > 0$, (c) and (d) for $\Delta m_{13}^2 < 0$. The other conditions are the same as in Fig. 2(a).

Comparing Fig. 3 (a) with (b) (or Fig. 3 (c) with (d)), we conclude that the appearance of the maximum for J_m/J at $E = O(10\text{MeV})$ depends on the sign of Δm_{12}^2 . Although the maximum appears in “neutrino” oscillation in the case of $\Delta m_{12}^2 > 0$, it appears in “anti-neutrino” oscillation in the case of $\Delta m_{12}^2 < 0$. Comparing Fig. 3 (a) with (c) (or Fig. 3 (b) with (d)), we conclude that the appearance of the maximum for J_m/J at $E = O(10\text{GeV})$ depends on the sign of Δm_{13}^2 . The maximum appears in “neutrino” oscillation in the case of $\Delta m_{13}^2 > 0$. On the other hand, it appears in “anti-neutrino” oscillation in the case of $\Delta m_{13}^2 < 0$.

These differences originate from the fact that the matter potential a for the maxima of J_m (see eqs. (21) and (27)) is proportional to Δm_{ij}^2 . If Δm_{ij}^2 is negative, then a is also negative and J_m has the maximum not in “neutrino” oscillation but in “anti-neutrino” oscillation. This is because the matter-modified Hamiltonian for anti-neutrino is obtained by replacing $a \rightarrow -a$.

6 Summary and Discussions

In this letter, we have studied matter modified Jarlskog factor J_m which appears in T violation for the lepton sector in neutrino oscillation. It was shown [12] that the inverse of J_m is proportional to the square root of a quartic polynomial of matter potential a from the relation $(\Delta_m)_{12}(\Delta_m)_{23}(\Delta_m)_{31}J_m = \Delta_{12}\Delta_{23}\Delta_{31}J$. We have presented the exact form of this polynomial with parameters in vacuum and have reconsidered the matter enhancement of J_m under the approximation $|\Delta m_{12}^2| \ll |\Delta m_{13}^2|$.

We show that J_m has (i) one maximum at $a = O(\Delta_{12})$ in the case of $\sin^2 2\theta_{13} \geq 1/9$ and (ii) two maxima at $a = O(\Delta_{12})$ and $a = O(\Delta_{13})$ in the case of $\sin^2 2\theta_{13} < 1/9$. Considering the constraint on $\sin^2 2\theta_{13}$ from the CHOOZ experiment, we conclude that the case (ii) is realized.

One maximum of J_m at $a = (\cos 2\theta_{12}/\cos^2 \theta_{13})\Delta_{12}$ is given by $J/\sin 2\theta_{12}$. J_m/J is roughly estimated as 12 for the SMA MSW, thus large enhancement is realized. The other maximum at $a = (1 - \frac{3}{2}\sin^2 2\theta_{13})\Delta_{13}$ is given by $|\Delta_{12}/\Delta_{13}|J/\sin 2\theta_{13}$ for $\sin^2 2\theta_{13} \ll 1/9$. If θ_{13} is small enough, the ratio J_m/J is enhanced. We have roughly estimated J_m/J as 4.2 for the LMA MSW solution at $\sin^2 2\theta_{13} = 4.0 \times 10^{-6}$.

In the case of $\sin^2 2\theta_{13} = 4.0 \times 10^{-6}$, our results agree with the results obtained by a different method [11]. Our results are also applicable around $\sin^2 2\theta_{13} \simeq 0.10$ which is the upper limit from the CHOOZ experiment.

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